Question 1 (10 Marks)

(a) If α, β and γ are the roots of the equation: $x^3 - 7x^2 + 17x - 15 = 0$, then find :

(i) $\sum_{i=1}^{3} c_i^3$ (ii) Solve the equation if $2 - i$ is a root   3 Marks

(b) Using mathematical induction to prove the validity of the following:

$1 \text{x} 2 + 2 \text{x} 5 + 3 \text{x} 8 + \cdots + n(3n - 1) = n^2(n + 1)$ 3 Marks

(c) Find the eigenvalues and the eigenvectors of the matrix: $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$ 4 Marks

Question 2 (10 Marks)

(a) Evaluate $(-2 + 2i)^{\frac{7}{3}}$ 3 Marks

(b) Find the sum to the first 20 terms of the series: $\sum_{r=1}^{n} \frac{1}{(r+3)(r+5)}$ 3 Marks

(c) Solve the following linear system using inverse matrix:

$2y - 2z + x = 5, \quad x - y - z = 0, \quad z + y + x - 6 = 0.$ 4 Marks

Question 3 (8 Marks)

(a) State the definition of the parabola.

(b) Show that the circles are orthogonal and find the points of intersection:

$(x - 1)^2 + (y - 1)^2 = 1, \qquad x^2 + y^2 + 2x - 2y - 1 = 0$

(c) Write the equation of ellipse where its vertices (2, 1), (2, 9) and foci (2, 2), (2, 8).

(d) Determine the type of the curve $x^2 - y^2 - 2x + y - 1 = 0$.

Question 4 (12 Marks)

(a) Determine center , vertices and sketch $4x^2 - y^2 + 16x + 4y + 4 = 0$.   3 Marks

(b) Determine the vertex, focus and sketch the parabola $y^2 - 8x - 4y - 4 = 0$.   3 Marks

(c) Write the equation of the plane that passes through the points: 2 Marks

(2, 0, 3), (− 1, 1, 0), (1, 2, 1).

(d) Find the angle between the plane: $x - 2y + 2z + 4 = 0$ and $\frac{x}{1} = \frac{y-2}{2} = \frac{z+1}{2}$.

Also, find the point of intersection.   2 Marks

(e) Write the name of each surface:

(i) $x^2 + y^2 - 2 = 0$ (ii) $y^2 = x^2 + z^2$ (iii) $x^2 + y^2 + z^2 - 5 = 0$ 2 Marks

Good Luck   Dr. Mona Samir   Dr. Mohamed Eid

Mathematics II Code: Math 102		Semester: Spring 2015
Mid-Term Exam: March 2015	**Modern University** For Technology & Information Faculty of Engineering	Time Allowed: 60 Minutes
Answer All questions		Total Mark: 30

Do not use Mobile as Calculator. Only use Calculator

Algebra: Answer in a separate paper

[1]Using mathematical induction to prove the validity of the following:

$$\frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \cdots \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$

[2] Solve the equation $x^3 - 8x^2 + 21x - 20$, if $(2 - i)$ is one of the roots.

[3] Find the sum to n terms of the series: $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots + \frac{1}{n\times(n+1)}$

[4] Divide $2x^5 + x - 3 - x^4 - 3x^3$ by $(x - 3)$, using Horner's method.

15

A. Geometry: Answer in a separate paper

[1] State the definition of the line.

2

[2] Find the radical axis and the points of intersection of the circles:
$$x^2 + y^2 - 1 = 0, \quad x^2 + y^2 - 2x + 2y + 1 = 0$$

3

[3] Find vertex, focus and sketch the parabola $x^2 + 4x + 8y - 28 = 0$.

3

[4] Write the equation of ellipse where its vertices $(1, 0), (1, 6)$ and foci $(1, 1), (1, 5)$

3

[5] Find center, vertices and sketch the hyperbola $x^2 - 4y^2 + 4x - 16y - 16 = 0$

4

Good luck   *Dr. Mona Samir*   *Dr. Mohamed Eid*

[1]State the definition of the plane.

[2]Write the equation of the plane that passes through the points $(1, -1, 0), (-1, 1, 2),$ $(3, 0, -1)$.

[3]Find the angle between the lines $\frac{x}{2} = \frac{y-1}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+1}{-2}$

[4]Determine the type of the curve $2x^2 + 3y^2 - 2x - 4y = 0$.

....

Answer

[1]State the definition of the parabola.

[2]Find the angle between the planes: $2x - 2y + z - 1 = 0, \quad 3x - 4z + 2 = 0$.

[3] Write the symmetric form and parametric form of the line which passes through

     $(1, -1, -2)$ and parallels to $\vec{U} = i - 2j + 3k$

[4]Determine the type of the curve $2x^2 - 3xy + 2y^2 - 16 = 0$.

....

Answer

[1]State the definition of the hyperbola.

[2]Write the equation of the plane that passes through the point $(2, -1, 3)$ and its normal vector is $\vec{U} = i - 2j + 3k$

[3]Find the angle between the plane: $2x - 2y + z - 1 = 0$ and the line $\dfrac{x-1}{1} = \dfrac{y}{-2} = \dfrac{z+1}{-2}$

[4]Determine the type of the curve $x^2 + 2xy + y^2 - 2x = 0$.

....

Answer

[1]State the definition of the circle.

[2]Find the angle between the plane: $x - y + 3z - 1 = 0$ and the line

   $x = 1,\ y = 2t - 1,\ z = t + 3.$

[3] Write the symmetric form and parametric form of the line that passes through the

   points $(0, 1, -3),\ (3, 2, 0).$

[4]Determine the type of the curve $x^2 - 4xy + 4y^2 - x + 3y - 2 = 0.$

....

Answer

[1]State the definition of the ellipse.

[2]Write the equation of the plane that passes through the points (1, 2, 0), (– 1, 1, 2), (3, 0, 3).

[3]Find the point of intersection of : $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+1}{2}$ and $x + 2y - z + 3 = 0$

[4]Determine the type of the curve $4x^2 - y^2 + 16x - 4y + 16 = 0$

....

Answer

[1] State the definition of the parabola.

[2] Find the equation plane that contains $\dfrac{x-3}{2} = \dfrac{y-1}{-1} = \dfrac{z+1}{-4}$ and $\dfrac{x-2}{1} = \dfrac{y+1}{-3} = \dfrac{z-2}{-1}$

[3] Write the equation of the plane that passes through the point $(1, -2, 3)$ and its

   normal vector joining the points $(1, -2, 3)$ and $(3, 2, 1)$

[4] Determine the type of the curve $x^2 - 3x + 2y + 1 = 0$

....

Answer

[1]State the definition of the line.

[2]Write the equation of the plane that passes through the point $(1, 1, 3)$ and its normal vector is $\vec{U} = 3i - 2k$

[3]Find the point of intersection of $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{-1}$ and $x - 2y + z + 1 = 0.$

[4]Determine the type of the curve $x^2 + 2xy + y^2 + x = 0$

....

Answer